3.3.59

The curve \( y = ax^2 + bx + c \) passes through the point \((1, 8)\) and is tangent to the line \( y = 6x \) at the origin. Find \( a \), \( b \) and \( c \).

Solution:

The curves contains point \((1, 8)\); therefore, \( 8 = a(1)^2 + b(1) + c \) or \( a + b + c = 8 \).

We also know that the curve is tangential to \( y = 6x \) at the origin; that implies that the curve goes though the origin; consequently, \( 0 = a(0)^2 + b(0) + c \) or \( c = 0 \). Since the the derivative at a point is equal to the slope of the tangent line at that point then, at \((0,0)\) the derivative is equal to 6, –that is the slope of the tangent line at that point (it is given, tangent line \( y = 6x \)).

Take the derivative, \( 2ax + b = 6 \) at the origin, where \( x = 0 \) implies that \( b = 6 \); and since \( a + b + c = 8 \), \( a = 2 \).

Answer: \( a = 2 \); \( b = 6 \) and \( c = 0 \).