**DEFINITION** Velocity (instantaneous velocity) is the derivative of position with respect to time. If a body's position at time $t$ is $s = f(t)$, then the body's velocity at time $t$ is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Besides telling how fast an object is moving along the horizontal line in Figure 3.12, its velocity tells the direction of motion. When the object is moving forward ($s$ increasing), the velocity is positive; when the object is moving backward ($s$ decreasing), the velocity is negative. If the coordinate line is vertical, the object moves upward for positive velocity and downward for negative velocity. The blue curves in Figure 3.13 represent position along the line over time; they do not portray the path of motion, which lies along the vertical $s$-axis.

If we drive to a friend's house and back at 30 mph, say, the speedometer will show 30 on the way over but it will not show -30 on the way back, even though our distance from home is decreasing. The speedometer always shows speed, which is the absolute value of velocity. Speed measures the rate of progress regardless of direction.

**DEFINITION** Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \frac{ds}{dt}$$

**EXAMPLE 2** Figure 3.14 shows the graph of the velocity $v = f'(t)$ of a particle moving along a horizontal line (as opposed to showing a position function $s = f(t)$ such as in Figure 3.13). In the graph of the velocity function, it's not the slope of the curve that tells us if the particle is moving forward or backward along the line (which is not shown in the figure), but rather the sign of the velocity. Looking at Figure 3.14, we see that the particle moves forward for the first 3 sec (when the velocity is positive), moves backward for the next 2 sec (the velocity is negative), stands motionless for a full second, and then moves forward again. The particle is speeding up when its positive velocity increases during the first second, moves at a steady speed during the next second, and then slows down as the velocity decreases to zero during the third second. It stops for an instant at $t = 3$ sec (when the velocity is zero) and reverses direction as the velocity starts to become negative. The particle is now moving backward and gaining in speed until $t = 4$ sec, at which time it achieves its greatest speed during its backward motion. Continuing its backward motion at time $t = 4$, the particle starts to slow down again until it finally stops at time $t = 5$ (when the velocity is once again zero). The particle now remains motionless for one full second, and then moves forward again at $t = 6$ sec, speeding up during the final second of the forward motion indicated in the velocity graph.