Answers to questions 14 to 22.

14. The velocity function, derivative of position function is \( v(t) = 14t + 4 \); therefore, \( v(t) = 0 \) at \( t = \frac{-2}{7} \) seconds; that is, during the time interval \([0,2]\) the particle does not change direction; then the displacement is given by position of particle at time two seconds minus position of particle at time 0 seconds: \( s(2) - s(0) = 45 - 9 = 36 \text{m} \).

Average velocity is displacement/time elapsed: \( \frac{36}{2} = 18 \text{ m/s} \).

15. Speed = \(|v(t)|\), and \( v(t) = 14t + 2 \), at the end of the time interval \( t = 2 \); therefore \( v(2) = 30 \text{ m/s} \), –notice that in this case speed = velocity since \( v(2) > 0 \). Acceleration, derivative of velocity function, \( a(t) = 14 \). That is, acceleration is constant, does not depend on time.

16. This is a graph of position with respect to time. The particle (body) moves forward whenever the velocity is positive; since velocity is the derivative of the position function, in those interval where the slope is positive, the velocity is positive, the particle is moving forward. That occurs from 0 to 1, from 3 to 4, from 5 to 6 and from 9 to 10.

17. This is a graph of velocity with respect to time. The acceleration is zero when velocity is constant. That occurs from time 2 to 3 and from 5 to 6.

18. A particle changes direction at time \( t \) when velocity is equal to zero, and acceleration is different from zero. Of course, when both, velocity and acceleration are equal to zero, the particle stops. In this case \( v(t) = 2t - 8 \), at \( t = 4 \), \( v(4) = 0 \), while \( a(2) = 2 \); that is, at \( t = 4 \) seconds the particle changes direction.

19. For the given position function \( v(t) = 3t^2 - 42t + 144 \) and \( a(t) = 6t - 42 \); factoring velocity function, \( v(t) = 3(t-8)(t-6) \); \( v(t) = 0 \), at \( t = 8 \) and \( t = 6 \), find \( a(6) = -6 \) and \( a(8) = 6 \).

20. In order to find the distance traveled by the particle we need to find when the particle change direction. Find \( v(t) = 3t^2 - 30t + 48 = 3(t-8)(t-2) \), that is, \( v(t) = 0 \) at \( t = 2 \) and \( t = 8 \). So, at time 2 seconds and 8 seconds the particle changes direction. The question asks for distance traveled from \( t = 0 \) to \( t = 3 \); so we need to find distance traveled from 0 to 2, and from 2 to 3.

Calculate position at \( t = 0 \), \( s(0) = 0 \); then \( s(2) = 44 \); at \( t = 3 \), the position is \( s(3) = 36 \). Let’s put these calculations in context: at \( t = 0 \), the particle is located at zero; after two seconds, reaches position 44 –the particle moved forward. Remember that at \( t = 2 \) it changes direction; therefore, it moves backwards, and at \( t = 3 \) is located at position 36. Total distance traveled: 44 units forward, and then from 44 to 36, 8 units backwards for a total of 52 total units –units in this problem are meters.

21. The body moves backwards in those time intervals in which velocity is negative. Let’s solve the inequality \( t^2 - 8t + 7 < 0 \), factors: \((t - 7)(t - 1) < 0\); create a sign chart using the number line: we find that from zero to 1, and from 7 to positive infinity \( v(t) > 0 \), or particle moves forward; from 1 to 7, \( v(t) < 0 \), that is, it moves backwards.
Big picture: from 0 to 1, the particle moves forward, at $t = 1$ stops and change direction; then from 1 to 7 it moves backwards, at $t = 7$ stops again and changes direction and moves forward.

22. Velocity increases when acceleration is positive. For the given $v(t)$, $a(t) = 2t - 6$, and $2t - 6 > 0$ for $t > 3$ secs.

Notes on graphs depicted on question 16 and 17:

16. Description:
This is a graph of position with respect to time. From $t = 1$ to $t = 2$ the particle moves forward, velocity is constant and it is equal to the slope of the line segment. At $t = 2$ the particle reaches position 2 and for a second, it does not move, velocity is zero. From $t = 2$ to $t = 3$ the particle moves backwards, velocity or slope of the line is negative, and at $t = 3$ returns to position 0. At $t = 3$ it changes direction, it moves again forward (velocity, slope of the position function is positive). At $t = 4$ reaches position 3, and changes direction again reaching position 0 at $t = 5$. Again the particle moves forward from $t = 5$ to 7, reaching position 4 at $t = 7$. At $t = 7$ it changes direction once again and this time around after one second or $t = 8$ crosses position zero and keeps moving backwards reaching position $-4$ at $t = 9$. Tired of going backwards, it changes direction and for another second, from 9 to 10, moves forward reaching position $-2$ at $t = 10$ and then disappears.

17. Description:
Graph of velocity with respect to time. From $t = 1$ to $t = 4$ the particle moves backwards, since velocities are negatives during this time interval. Specifically, from $t = 1$ to $t = 2$ the velocity decreases, but the speed, absolute value of velocity, increases; that is, the particle speeds up. Notice that during this interval the acceleration is negative. At $t = 2$ velocity is equal to $-2$, and for a second, from $t = 2$ to $t = 3$, there is no acceleration and both speed and velocity remain constant. From $t = 3$ to $t = 4$, velocity increases, speed decreases, acceleration is positive, the particle slows down reaching velocity and speed of zero at $t = 4$. Since velocity is zero at $t = 4$, but acceleration is not equal to zero (the slope of the line at that point is positive) the particle changes direction, moves forwards, velocity turns positive, speed also increases; that is, again the particle speeds up. From $t = 5$ to $t = 6$ acceleration is zero, velocity is constant and equal to 2. Form $t = 6$ to $t = 7$, velocity and speed decreases, acceleration is negative, the particle slows down reaching velocity equal zero at $t = 7$, that is, the particle stops, and at $t = 7$ the particle speeds up, move forward again, acceleration becomes is positive and at $t = 9$ disappears in space.