1) From a thin piece of cardboard 20 in. by 20 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

2) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 52 ft³. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

3) Determine the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 3.

4) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 96 in. Suppose you want to mail a box with square sides so that its dimensions are h by h by w and it's girth is 2h + 2w. What dimensions will give the box its largest volume?

5) The strength S of a rectangular wooden beam is proportional to its width times the square of its depth. Find the dimensions of the strongest beam that can be cut from a 14-in.-diameter cylindrical log. (Round answers to the nearest tenth.)

![Diagram of a semicircle with dimensions labeled d and w.]

Solve the problem.

6) You are planning to close off a corner of the first quadrant with a line segment 25 units long running from (x, 0) to (0, y). Show that the area of the triangle enclosed by the segment is largest when x = y.

7) How close does the curve \( y = \sqrt{x} \) come to the point \( \left( \frac{3}{2}, 0 \right) \)? (Hint: minimize the square of the distance).

8) How close does the semicircle \( y = \sqrt{16 - x^2} \) come to the point \( (1, \sqrt{2}) \)?

9) The function \( y = \cot x - \frac{2\sqrt{3}}{3} \csc x \) has an absolute maximum value on the interval \( 0 < x < \pi \). Find it.

10) Show that \( g(x) = \frac{a + x}{\sqrt{b^2 + (a + x)^2}} \) is an increasing function of x.
1) 13.3 in. × 13.3 in. × 3.3 in.; 592.6 in³
2) 4.7 ft × 4.7 ft × 2.4 ft
3) \( h = \frac{3\sqrt{2}}{2}, \ w = 3\sqrt{2} \)
4) \( \frac{64}{3} \) in. × \( \frac{64}{3} \) in. × 16 in.
5) \( w = 8.1 \) in.; \( d = 11.4 \) in.
6) If \( x, y \) represent the legs of the triangle, then \( x^2 + y^2 = 25 \).
   Solving for \( y \), \( y = \sqrt{625 - x^2} \)
   \( A(x) = xy = x\sqrt{625 - x^2} \)
   \( A'(x) = \frac{-x^2}{2\sqrt{625 - x^2}} + \frac{\sqrt{625 - x^2}}{2} \)
   Solving \( A'(x) = 0 \), \( x = \pm \frac{25\sqrt{2}}{2} \)
   Substitute and solve for \( y \):
   \( \left( \frac{25\sqrt{2}}{2} \right)^2 + y^2 = 625 \) ; \( y = \frac{25\sqrt{5}}{2} \) \( . : x = y \).
7) The distance is minimized when \( x = 1 \); the minimum distance is \( \sqrt{\frac{5}{4}} \) units.
8) \( x = 2.27 \)
9) \( y' = -csc^2(x) + \frac{2\sqrt{3}}{3}csc(x)\cot(x) = csc(x)\left[ \frac{2\sqrt{3}}{3}\cot(x) - csc(x) \right] = 0 \Rightarrow \left[ \frac{2\sqrt{3}}{3}\cot(x) - csc(x) \right] = 0 \Rightarrow 1 - \frac{2\sqrt{3}}{3}\cos(x) = 0 \Rightarrow \cos(x) = \frac{1}{2\sqrt{3}} \Rightarrow x = \frac{\pi}{6} \)
10) Notice that \( g'(x) = \frac{b^2}{(b^2 + (a + x)^2)^{3/2}} \) is positive for all values of \( x \). Therefore \( g \) is increasing everywhere.