19. "Amount of material": Surface Area

\[ V = 40 \text{ ft}^3 \text{ given.} \]

\[ V = lwh \]

\[ l = 2w \]

\[ \Rightarrow \text{"it is twice as long as it is wide".} \]

Total Surface Area: \[ SA = 2lh + 2hw + lw \]

Since \[ l = 2w \] and \[ V = lwh \]

\[ 40 = 2w \cdot w \cdot h \]

\[ \therefore h = \frac{20}{w^2} \]

\[ SA = 2\left(\frac{20}{w^2}\right) + 2\left(\frac{20}{w^2}\right)w + 2w \cdot w \]

\[ \Rightarrow SA = \frac{40}{w} + \frac{40}{w} + 2w^2 \]

\[ \Rightarrow SA = \frac{120}{w} + 2w^2 \]

\[ \text{In order to minimize } SA, \text{ take derivative } = 0. \]

\[ SA' = -\frac{120}{w^2} + 4w \]

\[ -120 + 4w^2 = 0 \]

\[ w = \sqrt{\frac{120}{4}} \]

\[ w = 3.11 \text{ ft.} \]

20. \[ A = xy^2 \] Maximize, given \[ x + y^2 = 4 \] constraint.

\[ \therefore y^2 = 4 - x \]

\[ A = x(4-x) = 4x - x^2 \]

\[ A' = 4 - 2x \]

\[ 4 - 2x = 0 \]

\[ \therefore x = 2 \]

\[ \text{Therefore } y^2 = 4 - x \]

\[ \therefore y = \sqrt{2} \]

\[ \text{Ans: } x = 2 \]

\[ y = \sqrt{2} \]
21)\[B \text{ base}\]

\[b + 2x = 12 \quad \therefore \quad b = 12 - 2x\]

Max capacity = Max cross section, given by Area of cross section. Notice that we cannot find volume since length is not given. Area of cross section, \(A\):

\[A = bx\]
\[A = (12 - 2x)x\]
\[A = 12x - 2x^2, \quad A' = 12 - 4x\]
\[12 - 4x = 0\]
\[\therefore \quad x = 3 \text{ inches}\]

22. Antiderivative of cosine is sine since derivative of sine is cosine. When we take derivative we multiply by derivative of the "inside" or function's argument, taking antiderivatives we divide by deriv of the argument. Answer is:

\[\frac{4 \sin 7x}{7} = \frac{4}{7} \sin 7x\]

23. Recall that derivative of \(\csc u = \quad = - \csc u \cdot \cot u \cdot u'\)

Therefore, the antiderivative of \(\csc u \cdot \cot u\) is \(-\csc u \div u' \) in this case:

\[-3 \csc 2x \quad = - \frac{3}{2} \csc 2x\]
24) ∫ inddefinite integral: anti-derivatives

\[ \int -6 \cos t \, dt = -6 \sin t + C \]

since anti-deriv of cosine is sine

25) \[
\int \frac{\sec \theta \cdot \cos \theta}{\sec \theta - \cos \theta} \, d\theta
\]

since \( \sec \theta = \frac{1}{\cos \theta} \) multiply top and bottom by \( \cos \theta \)

\[
= \int \frac{1}{1 - \cos^2 \theta} \, d\theta
\]

\[
= \int \frac{1}{\sin \theta} \, d\theta = \int \csc^2 \theta \, d\theta.
\]

now, recall that deriv of \( \cot \theta \) is \( -\csc^2 \theta \); therefore:

\[ \int \csc^2 \theta \, d\theta = -\cot \theta + C \]

26) \[
\int 8e^{3x} - 7e^{-x} \, dx
\]

recall: deriv of \( y = e^{bx} \)

\[ y' = be^{bx} \]

therefore, the antiderivative of \( e^{bx} \) is \( \frac{1}{b} e^{bx} \).

\[ = \frac{8}{3} e^{3x} + 7e^{-x} + C \]

\[ = \frac{8}{3} e^{3x} + 7e^{-x} + C \]

\[ = \frac{8}{3} e^{3x} + 7e^{-x} + C \]
27. Anti-derivative: undo derivative; therefore, anti-derivative of the 2nd derivative yields the first derivative. Also \( \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \)

\[ \therefore \int 4 - 7x \, dx = 4x - 7\frac{x^2}{2} + C \]

\[ \therefore y' = 4x - 7\frac{x^2}{2} + C \quad \text{given that} \quad y'(0) = 8 \]

\[ \therefore C = 8 \]

\[ y' = 4x - 7\frac{x^2}{2} + 8 \]

Anti-derivative of the 1st derivative yields original function + C.

\[ \int 4x - 7\frac{x^2}{2} + 8 \, dx = \frac{4x^2}{2} - \frac{7}{3}x^3 + 8x + C \]

\[ \therefore y = f(x) = 2x^2 - \frac{7}{3}x^3 + 8x + C \quad \text{given that} \quad y(0) = 2 \]

\[ \therefore C = 2 \]

\[ y = f(x) = 2x^2 - \frac{7}{3}x^3 + 8x + 2 \]

28. \( \int \frac{1}{x+6} \, dx = \ln |x+6| + C \)

\[ y = \ln |x+6| + C \quad \text{given} \quad y(-5) = 8 \]

\[ y(-5) = \ln |(-5)+6| + C = 8 \]

\[ y(-5) = \ln 1 + C = 8 \]

\[ \therefore C = 8 \]

\[ = \ln (x+6) + 8 \]
29. Recall that "derivative of position with respect to time" = velocity. And, derivative of velocity with respect to time = acceleration.

We undo derivatives by taking anti-derivatives or integrals.

Integrated of acceleration = velocity
Integrated of velocity = position with respect to time.

Given: \( a = 16 \cos 4t \)

\[ v = \int 16 \cos 4t \, dt = \frac{16 \sin 4t}{4} + C \]

\[ v = 4 \sin 4t + C \quad \text{since} \quad v(0) = -8 \]

\[-8 = 4 \sin (0) + C \]

\[-8 = C \quad \therefore \quad v = 4 \sin 4t - 8 \]

\[ s = \int \text{velocity} \, dt \quad \therefore \quad s = \int 4 \sin 4t - 8 \, dt \]

\[ s = \frac{-4 \cos 4t}{4} - 8t + C \]

\[ s = -\cos 4t - 8t + C \quad \text{given} \quad s(0) = 3 \]

\[ 3 = -\cos (0) - 8(0) + C \]

\[ 3 = -1 + C \quad \therefore \quad C = 4 \]

\[ s(t) = -\cos 4t - 8t + 4 \]

30. \( \frac{d^2 s}{dt^2} \)

Second derivative of position = acceleration.

\[ v = \int -5.2 \, dt = -5.2t + C \quad \text{since object is 'dropped'} \]

\[ v = -5.2t \quad \therefore \quad s(t) = \int -5.2t \, dt = -5.2\frac{t^2}{2} + C \]

\[ C = 0 \]

\[ v = -5.2t \quad \therefore \quad s(t) = -5.2t^2 + 10 \quad \text{initial position, given} = 10. \]

When object hits surface \( s(t) = 0 \) \{position is zero\}; solve for \( t \), set \( s(t) = 0 \):

\[ t = \sqrt{\frac{2\times10}{5.2}} \approx 1.96 \text{ seconds.} \]