Practice 05

2. Use point-slope formula: \( y - y_1 = m(x - x_1) \) given point \((6, 54)\), that is \((x_1, y_1)\) we need \(m\), the slope. Recall that \(m = f'(x_1)\).

In words: slope of tangent line at a point is equal to the derivative evaluated at given \(x\) value.

For \(y = \frac{x^3}{4}\) find \(y' = f'(x) = \frac{3}{4}x^2\) \(x_1 = 6\).

\[
\therefore f'(6) = \frac{3}{4}(6)^2 = 27
\]
now using \(y - y_1 = m(x - x_1)\)

\[y - 54 = 27(x - 6)\]

\[\therefore y = 27x - 108\]

3. For \(y = x^2 + 4\) at \((2, 0)\)

\[m = f'(x) = y' = 2x \quad \therefore m = 2(2) = 4\]

\[y - y_1 = m(x - x_1) \implies y - 0 = 4(x - 2)\]

\[y = 4x - 8\]

4. Just finding slope. Again, \([m = f'(x)]\)

\[f'(x) = 2x - 8 \quad \text{at} \quad x = 1 \quad m = f'(1) = 2(1) - 8 = -6\]

5. \(y = \frac{4}{6 + x}\) Let's find derivative using

\[f'(x) = \lim_{c \to x} \frac{f(c) - f(x)}{c - x}\]

see next page...
\[ f'(x) = \frac{4}{6+c} - \frac{4}{6+x} = -\lim_{c \to x} \frac{4[(6+x) - (6+c)]}{(6+c)(6+x)} \]

\[ = \lim_{c \to x} \frac{4(c-x)}{(6+c)(6+x)} = \lim_{c \to x} \frac{4(c-x)}{(6+x)(6+x)} \]

Now, notice that \( x - c = -(c-x) \)

Rewrite:

\[ \lim_{c \to x} \frac{-(c-x)}{(6+c)(6+x)} = \lim_{c \to x} \frac{-4}{(6+c)(6+x)} \]

As \( c \to x \), \( c \) becomes \( x \):

\[ f'(x) = \frac{-4}{(6+x)(6+x)} \]

\[ f'(x) = \frac{-4}{(6+x)^2} \]

At \( x = 5 \):

\[ f'(5) = \frac{-4}{(6+5)^2} = -\frac{4}{121} \]

6. Horizontal tangents mean that the slope of the tangent line is zero. Therefore, we need to find the derivative and set it equal to zero and solve for \( x \):

\[ f'(x) = 4x + 4 \implies 4x + 4 = 0, \quad x = -1 \]

Question asks for \((x, y)\), the point.

\( x = -1, \quad y = 2(-1)^2 + 4(-1) + 3 = 1 \)

Point is \((-1, 1)\)
6. Slope is \(-1\), then, since \(m = f'(x)\) find derivative, set \(c = m\), \(-1\) in this case and solve for \(x\):

\[
f(x) = 4x^{-1}; \quad f'(x) = -4x^{-2} = -\frac{4}{x^2}
\]

\[-\frac{4}{x^2} = -1 \iff x^2 = 4 \iff x = \pm 2.
\]

For \(x_1 = 2\), \(y_1 = \frac{4}{2} = 2\) \((2, 2)\)

For \(x_2 = -2\), \(y_2 = \frac{4}{-2} = -2\) \((-2, -2)\)

Recall that slope is given and equal to \(-1\).

Using \(y - y_1 = m(x - x_1)\)

1st point \((2, 2)\) \(\therefore y - 2 = -1(x - 2) \Rightarrow y = -x + 4\)

2nd point \((-2, -2)\) \(\therefore y + 2 = -1(x + 2) \Rightarrow y = -x - 4\)

8. This time \(m = 2\). Find \(f'(x)\) and set \(c = 2\).

\[
f'(x) = 4x - 2 \Rightarrow 4x - 2 = 2 \iff x = 1
\]

At \(x = 1\), \(y_1 = f(1) = 2(1)^2 - 2(1) + 1\)

Point is \((1, 1)\)

Using \(y - y_1 = m(x - x_1)\)

\[y - 1 = 2(x - 1) \Rightarrow y = 2x - 1\]
9. Speed is abs value of velocity.
   Velocity is the derivative of the position function.
   \[ v = s(t) = 16.86t \]
   Speed = \( |v| = 16.86 \times 3 = 50.58 \text{ m/s} \).

10. \( f'(x) = 5 \); \( f'(2) = 5 \)

11. \( g'(x) = 3x^2 + 5 \); \( g'(1) = 8 \)

12. \( f(x) = 8x^3 \); \( f'(x) = 24x^2 \); \( f'(-1) = -8 \)

13. \( v = t + 9t^{-1} \)
    \[ \therefore \frac{dv}{dt} = v' = 1 - 9(t^{-2}) = 1 - \frac{9}{t^2}. \]

14. \[ m = f(4) \]
    \[ f'(4) = -20t^3 - 6t^2 \text{ at } t = -1 \]
    \[ m = f'(-1) = -20(-1)^3 - 6(-1)^2 = 20 - 6 = 14 \]

15. \[ f(x) = \lim_{c\to x} \frac{1}{c+x} = \lim_{c\to x} \frac{4(x-c)}{(c+7)(x+7)} \]
    \[ = \lim_{c\to x} \frac{4(x+c)}{(c+7)(x+7)} \]
    \[ = \lim_{c\to x} \frac{4(x-c)}{(c+7)(x+7)} \]
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\[
\lim_{c \to x} \frac{-4(c-x)}{(c+7)(x+7)} = \frac{-4}{(x+7)^2}
\]

since \(c \to x\) we write:

\[
\lim_{c \to x} \frac{-4}{(x+7)^2} = \frac{-4}{(x+7)^2}
\]

\[f'(x) = \frac{-4}{(x+7)^2}\]

\[\text{(b)} \lim_{c \to x} \frac{(5c^2 - 9c + 5) - (5x^2 - 9x + 5)}{c-x} \quad \text{rearrange}\]

\[
\lim_{c \to x} \frac{5c^2 - 5x^2 - 9c + 9x}{c-x} = \lim_{c \to x} \frac{5(c^2 - x^2) - 9(c-x)}{(c-x)} = \lim_{c \to x} \frac{5(c+x)(c-x) - 9(c-x)}{(c-x)}
\]

\[= \lim_{c \to x} \frac{5(c+x)(c-x) - 9(c-x)}{(c-x)} \quad \text{as } c \to x \text{ replace } c \text{ by } x\]

\[= 5(x+x) - 9 = 5(2x) - 9 = 10x - 9\]

\[f'(x) = 10x - 9\]
\( \lim_{c \to x} \frac{4c + \sqrt{c} - (4x + \sqrt{x})}{c - x} = \lim_{c \to x} \frac{4c - 4x + \sqrt{c} - \sqrt{x}}{c - x} \)

\[ = \lim_{c \to x} \frac{4(c-x) + \sqrt{c} - \sqrt{x}}{c - x} \quad \text{mult by conjugate:} \]

\[ = \lim_{c \to x} 4 + \frac{(\sqrt{c} - \sqrt{x}) \cdot (\sqrt{c} + \sqrt{x})}{c - x} \]

\[ = \lim_{c \to x} 4 + \frac{(c-x)}{(c-x)(\sqrt{c} + \sqrt{x})} \quad \text{and as } c \to x \]

\[ \therefore = 4 + \frac{1}{\sqrt{x} + \sqrt{x}} = 4 + \frac{1}{2\sqrt{x}} \]

18) see ans sheet for graph.

19) the function is continuous on the interval \((-\infty, x)\), but it has two "corners", at \(x = -2\), and \(x = 2\). not continuous at corners.

20. To the left of zero, \(f(0^-) = 5\)
To the right of zero, \(f(0^+) = 5\).
the function is not continuous at zero; therefore it is not differentiable at zero.
21. Again, not differentiable at $(0,0)$; it is a "corner": approaching zero from the right the slope is 2 [given function is $y=2x$] approaching zero from the left the slope is 1 [given function is $y=x$] to be differentiable the slope to the tangents line at the given point exists and it is unique. —